

Appendix A

Diagram for the computation by the Engine of the Numbers of Bernoulli. See Note G. (page 722 *et seq.*)

Number of Operation.	Nature of Operation.	Variables acted upon.	Variables receiving results.	Indication of change in the value on any Variable.	Statement of Results.	Data.										Working Variables.										Result Variables.									
						$1V_1$	$1V_2$	$1V_3$	$1V_4$	$1V_5$	$1V_6$	$1V_7$	$1V_8$	$1V_9$	$1V_{10}$	$1V_{11}$	$1V_{12}$	$1V_{13}$	$1V_{14}$	$1V_{15}$	$1V_{16}$	$1V_{17}$	$1V_{18}$	$1V_{19}$	$1V_{20}$	$1V_{21}$	$1V_{22}$	$1V_{23}$	$1V_{24}$	$1V_{25}$					
						1	2	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
						1	2	n																											
1	\times	$1V_2 \times 1V_3$	$1V_4, 1V_5, 1V_6$	$1V_2 = 1V_2$ $1V_3 = 1V_3$ $1V_4 = 2V_4$ $1V_5 = 1V_5$ $1V_6 = 1V_6$	$= 2n$...	2	n	2n	2n	2n																								
2	$-$	$1V_4 - 1V_1$	$2V_4$	$1V_4 = 2V_4$ $1V_1 = 1V_1$ $1V_2 = 1V_2$	$= 2n - 1$...	1	...	2n - 1																										
3	$+$	$1V_5 + 1V_1$	$2V_5$	$1V_5 = 2V_5$ $1V_1 = 1V_1$ $1V_2 = 1V_2$	$= 2n + 1$...	1	...	2n + 1																										
4	$+$	$2V_5 + 2V_4$	$1V_{11}$	$2V_5 = 0V_5$ $2V_4 = 0V_4$ $1V_{11} = 2V_{11}$	$= 2n - 1$...	1	...	0	0																									
5	$+$	$1V_{11} + 1V_2$	$2V_{11}$	$1V_{11} = 2V_{11}$ $1V_2 = 1V_2$ $1V_3 = 1V_3$	$= \frac{1}{2} \cdot 2n + 1$...	2	...																											
6	$-$	$1V_{13} - 2V_{11}$	$1V_{13}$	$2V_{11} = 0V_{11}$ $1V_{13} = 1V_{13}$ $1V_2 = 1V_2$	$= -\frac{1}{2} \cdot 2n - 1 = A_0$																												
7	$-$	$1V_8 - 1V_1$	$1V_{10}$	$1V_8 = 1V_8$ $1V_1 = 1V_1$ $1V_2 = 1V_2$	$= n - 1 (= 3)$...	1	...	n																										
8	$+$	$1V_2 + 0V_7$	$1V_7$	$1V_2 = 1V_2$ $0V_7 = 1V_7$ $1V_3 = 1V_3$	$= 2 + 0 = 2$...	2																												
9	$+$	$1V_6 + 1V_2$	$2V_{11}$	$1V_6 = 1V_6$ $1V_2 = 1V_2$ $1V_3 = 1V_3$	$= \frac{2n}{2} = A_1$			2n																									
10	\times	$1V_{21} \times 2V_{11}$	$1V_{12}$	$1V_{21} = 1V_{21}$ $2V_{11} = 1V_{12}$ $1V_2 = 1V_2$	$= B_1 \cdot \frac{2n}{2} = B_1 A_1$																												
11	$+$	$1V_{12} + 1V_{13}$	$2V_{13}$	$1V_{12} = 0V_{12}$ $1V_{13} = 1V_{13}$ $1V_2 = 1V_2$	$= -\frac{1}{2} \cdot 2n - 1 + B_1 \cdot \frac{2n}{2}$																												
12	$-$	$1V_{10} - 1V_1$	$2V_{10}$	$1V_{10} = 2V_{10}$ $1V_1 = 1V_1$ $1V_2 = 1V_2$	$= n - 2 (= 2)$...	1	...																											
13	$-$	$1V_6 - 1V_1$	$2V_6$	$1V_6 = 2V_6$ $1V_1 = 1V_1$ $1V_2 = 1V_2$	$= 2n - 1$...	1	...																											
14	$+$	$1V_1 + 1V_7$	$2V_7$	$1V_1 = 1V_1$ $1V_7 = 1V_7$ $1V_2 = 1V_2$	$= 2 + 1 = 3$...	1	...																											
15	$+$	$2V_6 + 2V_7$	$1V_8$	$2V_6 = 2V_6$ $2V_7 = 2V_7$ $1V_8 = 1V_8$	$= \frac{2n - 1}{3}$			2n - 1																									
16	\times	$1V_8 \times 2V_{11}$	$1V_{11}$	$1V_8 = 1V_8$ $2V_{11} = 1V_{11}$ $1V_2 = 1V_2$	$= \frac{2n - 1}{3}$																												
17	$-$	$2V_6 - 1V_1$	$2V_6$	$2V_6 = 2V_6$ $1V_1 = 1V_1$ $1V_2 = 1V_2$	$= 2n - 2$...	1	...																											
18	$+$	$1V_1 + 2V_7$	$2V_7$	$1V_1 = 1V_1$ $2V_7 = 2V_7$ $1V_2 = 1V_2$	$= 3 + 1 = 4$...	1	...																											
19	$+$	$2V_6 + 2V_7$	$1V_9$	$2V_6 = 2V_6$ $2V_7 = 2V_7$ $1V_9 = 1V_9$	$= \frac{2n - 2}{4}$			2n - 2																									
20	\times	$1V_9 \times 1V_{11}$	$1V_{11}$	$1V_9 = 1V_9$ $1V_{11} = 1V_{11}$ $1V_2 = 1V_2$	$= \frac{2n - 2}{3} \cdot \frac{2n - 1}{3} = A_3$																												
21	\times	$1V_{22} \times 1V_{11}$	$1V_{12}$	$1V_{22} = 1V_{22}$ $1V_{11} = 1V_{11}$ $1V_2 = 1V_2$	$= B_3 \cdot \frac{2n - 2}{3} \cdot \frac{2n - 1}{3} = B_3 A_3$																												
22	$+$	$2V_{12} + 2V_{13}$	$2V_{13}$	$2V_{12} = 0V_{12}$ $2V_{13} = 2V_{13}$ $1V_2 = 1V_2$	$= A_3 + B_1 A_1 + B_3 A_3$																												
23	$-$	$2V_{10} - 1V_1$	$2V_{10}$	$2V_{10} = 2V_{10}$ $1V_1 = 1V_1$ $1V_2 = 1V_2$	$= n - 3 (= 1)$...	1	...																											
Here follows a repetition of Operations thirteen to twenty-three.																																			
24	$+$	$1V_{13} + 0V_{24}$	$1V_{24}$	$1V_{13} = 1V_{13}$ $0V_{24} = 1V_{24}$ $1V_2 = 1V_2$	$= B_7$																												
25	$+$	$1V_1 + 1V_8$	$1V_8$	$1V_1 = 1V_1$ $1V_8 = 1V_8$ $1V_2 = 1V_2$	$= n + 1 = 4 + 1 = 5$...	1	...	n + 1																										

Fig. 1. Diagram of an algorithm for the Analytical Engine for the computation of Bernoulli numbers, from "Sketch of The Analytical Engine Invented by Charles Babbage" by Luigi Menabrea. 1842. Wikimedia Commons. Accessed July 1, 2019.

Diagram for the co

Number of Operation.	Nature of Operation.	Variables acted upon.	Variables receiving results.	Indication of change in the value on any Variable.	Statement of Results.
1	\times	$1V_2 \times 1V_3$	$1V_4, 1V_5, 1V_6$	$1V_2 = 1V_2$ $1V_3 = 1V_3$ $1V_4 = 2V_4$ $1V_5 = 1V_5$ $1V_6 = 1V_6$	$= 2n$
2	$-$	$1V_4 - 1V_1$	$2V_4$	$1V_4 = 2V_4$ $1V_1 = 1V_1$ $1V_2 = 1V_2$	$= 2n - 1$
3	$+$	$1V_5 + 1V_1$	$2V_5$	$1V_5 = 2V_5$ $1V_1 = 1V_1$ $1V_2 = 1V_2$	$= 2n + 1$
4	$+$	$2V_5 + 2V_4$	$1V_{11}$	$2V_5 = 0V_5$ $2V_4 = 0V_4$ $1V_{11} = 2V_{11}$	$= 2n - 1$
5	$+$	$1V_{11} + 1V_2$	$2V_{11}$	$1V_{11} = 2V_{11}$ $1V_2 = 1V_2$ $1V_3 = 1V_3$	$= \frac{1}{2} \cdot 2n + 1$
6	$-$	$1V_{13} - 2V_{11}$	$1V_{13}$	$2V_{11} = 0V_{11}$ $1V_{13} = 1V_{13}$ $1V_2 = 1V_2$	$= \frac{1}{2} \cdot 2n - 1$
7	$-$	$1V_8 - 1V_1$	$1V_{10}$	$1V_8 = 1V_8$ $1V_1 = 1V_1$ $1V_2 = 1V_2$	$= n - 1 (= 3)$
8	$+$	$1V_2 + 0V_7$	$1V_7$	$1V_2 = 1V_2$ $0V_7 = 1V_7$ $1V_3 = 1V_3$	$= 2 + 0 = 2$
9	$+$	$1V_6 + 1V_2$	$2V_{11}$	$1V_6 = 1V_6$ $1V_2 = 1V_2$ $1V_3 = 1V_3$	$= \frac{2n}{2} = A_1$
10	\times	$1V_{21} \times 2V_{11}$	$1V_{12}$	$1V_{21} = 1V_{21}$ $2V_{11} = 1V_{12}$ $1V_2 = 1V_2$	$= B_1 \cdot \frac{2n}{2} = B_1 A_1$
11	$+$	$1V_{12} + 1V_{13}$	$2V_{13}$	$1V_{12} = 0V_{12}$ $1V_{13} = 1V_{13}$ $1V_2 = 1V_2$	$= -\frac{1}{2} \cdot 2n - 1 + B_1 \cdot \frac{2n}{2}$
12	$-$	$1V_{10} - 1V_1$	$2V_{10}$	$1V_{10} = 2V_{10}$ $1V_1 = 1V_1$ $1V_2 = 1V_2$	$= n - 2 (= 2)$

Fig. 2. Close-up of top left corner of Note G (Fig. 1), showing the first two rows of algorithmic equations. The values in the third column on the table show the inputs and operation, while the values in the fourth column show the respective output. Note how the subscripts of each variable (V), designating an arm of the Analytical Engine, show up in consequent operations (in the third column), and also how the superscripts of each variable change when an operation inscribes a new value.